Sources of Phase Noise and Jitter in Oscillators

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The output signal of an oscillator, no matter how good it is, will contain all kinds of unwanted noises and signals. Some of these unwanted signals are spurious output frequencies, harmonics and sub-harmonics, to name a few. The noise part can have a random and/or deterministic noise in both the amplitude and phase of the signal. Here we will look into the major sources of some of these unwanted signals/noises.

Oscillator noise performance is characterized as jitter in the time domain and as phase noise in the frequency domain. Which one is preferred, time or frequency domain, may depend on the application. In radio frequency (RF) communications, phase noise is preferred while in digital systems, jitter is favored. Hence, an RF engineer would prefer to address phase noise while a digital engineer wants jitter specified.

Note that phase noise and jitter are two linked quantities associated with a noisy oscillator and, in general, as the phase noise increases in the oscillator, so does the jitter.

The best way to illustrate this is to examine an ideal signal and corrupt it until the signal starts resembling the real output of an oscillator.

The Perfect or Ideal Signal

An ideal signal can be described mathematically as follows:

$$V(t) = A_0 \sin(2\pi f_0 t)$$

Equation 1

Where:
A_0 = nominal peak voltage
f_0 = nominal fundamental frequency
t = time

A representation of this ideal signal in both the frequency and time domain is illustrated in Figure 1.

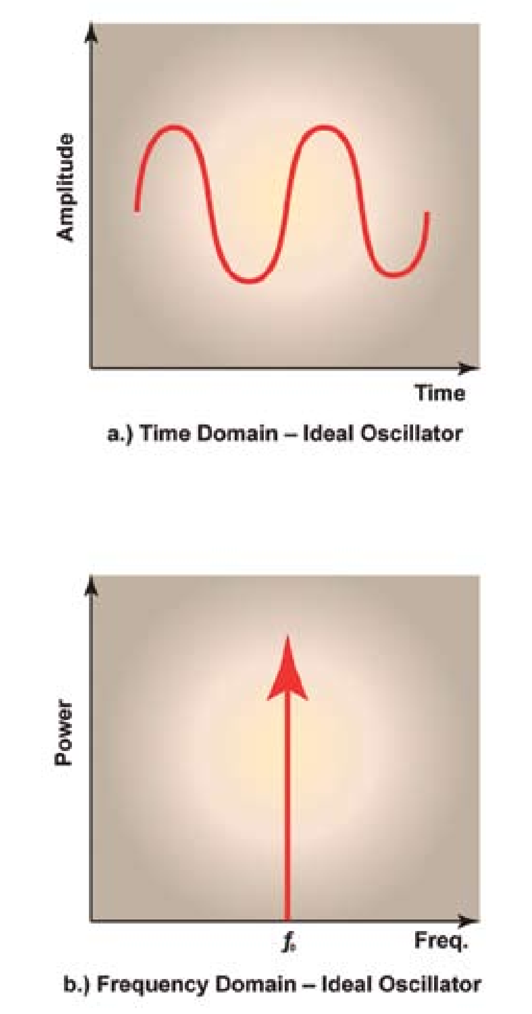

Figure 1

Now, if we add some amplitude noise to this equation, we obtain:

$$V(t) = [A_0 + \varepsilon(t)]\sin(2\pi f_0 t)$$

Equation 2

Where: $\varepsilon(t)$ = random deviation of amplitude from nominal- "AM noise"

Let's make things more interesting by adding a random phase component to Equation 2. This gives us:

$$V(t) = [A_0 + \varepsilon(t)]\sin[2\pi f_0 t + \Delta\phi(t)]$$

Equation 3

Where: $\Delta\phi(t)$ = random deviation of phase from nominal- "phase noise"

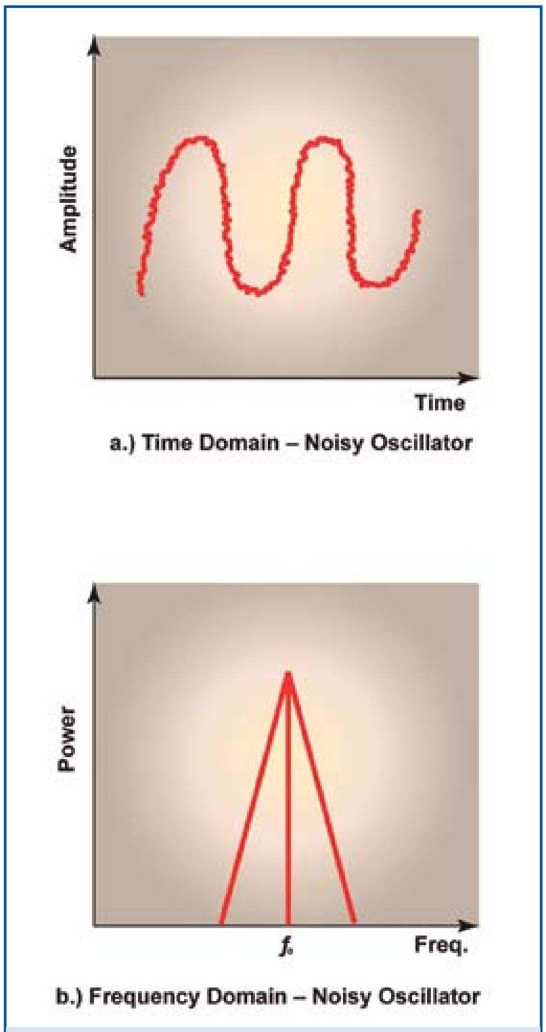

Figure 2

The new time and frequency domain representation is shown in Figure 2 while a vector representation of Equation 3 is illustrated in Figure 3 (a and b.)

It turns out that oscillators are usually saturated in amplitude level and therefore we can neglect the AM noise in Equation 3. Hence we can go back and simplify the equation to:

$$V(t) = A_0 \sin[2\pi f_0 t + \Delta\phi(t)]$$

Equation 4

Let us further expand our equation by adding a deterministic component to the phase. Our equation now becomes,

$$V(t) = A_0 \sin[2\pi f_0 t + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$

Equation 5

Where: m_d is the amplitude of the deterministic signal which is phase modulating the carrier and f_d is the frequency of the deterministic signal.

Equation 5 cannot be simplified by ordinary trigonometry, but can be expressed as a series of sinusoids by using the Bessel functions of the first kind. However, this is not necessary for our discussion and is beyond the scope of this exercise.

Now imagine adding all the harmonics and any sub-harmonics to our signal. The

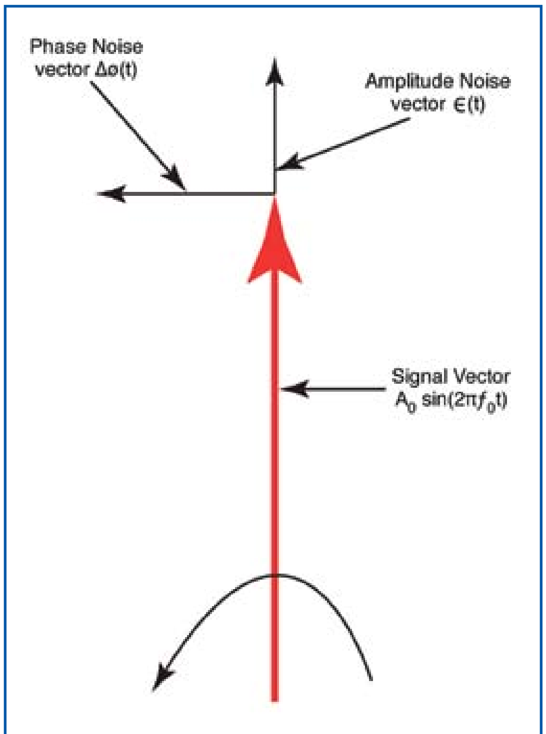

Fig. 3A Vector Representation of $V(t) = [A_0 + \epsilon(t)] \cdot \sin[2\pi f_0 t + \Delta\phi(t)]$ with Є(t) and Δø(t) fixed.

Figure 3A

equation can grow very quickly as below:

$$V(t) = A_0 \sin[2\pi f_0 t + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$
$$+ A_1 \sin[2\pi 2 f_0 t + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$
$$+ A_2 \sin[2\pi 3 f_0 t + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$
$$\cdot$$
$$\cdot$$
$$+ A_{N_{th}} \sin[2\pi N_{th} f_0 t + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$
$$+ A_{sub} \sin[2\pi (\frac{f_0}{N}) + \Delta\phi(t) + m_d \sin(2\pi f_d t)]$$

Equation 6

Where the last term represents one sub-harmonic. You can continue to expand the equation and add terms for the spurious outputs but we will stop here and now explain where some of these unwanted signals and noises come from in an oscillator.

As can be seen from Equation 6, we have a very complex signal coming out of the oscillator with random and deterministic phase variations, harmonics and sub-harmonics, etc. Where are all these signals and noise sources coming from?

Random Noise Sources

The random components are contributed by, but not limited to, the three major sources of noise. These are:

1. **"Thermal Noise"** - kTB noise caused by the Brownian motion of electrons due to thermal agitations. It increases with temperature, bandwidth and noise resistance.
2. **"Shot Noise"** - caused by discontinuous current (holes and electrons) flow across pn junction potential barriers.
3. **"Flicker Noise"** - noise that is spectrally related to 1/f and found in all active devices, as well as some passive components such as carbon resistors.

The close in phase noise of oscillator is directly proportional to the frequency determining resonators' Q. The higher the Q of the resonator, the lower the close in phase noise. Far out noise (the noise floor) comes from all the contribution of all circuitry in the oscillator. We will call this white noise.

Deterministic Sources

The deterministic component can come from but is not limited to the following four sources. (See Figure 4 for a general representation of the typical output spectra of a real oscillator.)

1. **Power supply feed-through (hum)** - If the power supply is not clean, any signal riding on it can get into the feedback path of the oscillator and possibly phase modulate the output frequency or carrier.
2. **Spurious signals** - An oscillator is designed to have just one feedback path to generate the desired output signal. However, the reality is that many such feedback paths exist, giving rise to spurious oscillations at many different frequencies and amplitudes.
3. **Quartz crystal resonator resonance** - In a quartz crystal resonator-based oscillator, crystal-controlled spurious outputs may be induced by one of the crystal's response/resonance that is not being utilized. For example, the strongest response is called the fundamental, there are odd overtone responses (i.e., 3rd, 5th, 7th, etc) and other so-called spurious that all exist in every quartz crystal. An oscillator can excite some of these creating a deterministic component in the output signal.
4. **Sub-harmonic(s)** - A sub-harmonic is an exact fraction of the output frequency. An oscillator output signal that is derived from a lower frequency source by some type of frequency multiplication will have at least one sub-harmonic. The sub-harmonic will contribute directly to the deterministic jitter of the output signal as well as harmonic distortion.

Harmonics Sources

Harmonics - can be generated by the non-linearity of any component in the oscillator circuitry, but usually the strongest contributors are any pn junction the signal must go through. A very far harmonic, for example the 11th harmonic can de-sensitize a receiver by falling into the IF or receiving frequency band. Harmonic distortion can be a problem if the signal output of the oscillator is going to be converted from a sine wave into a square wave.

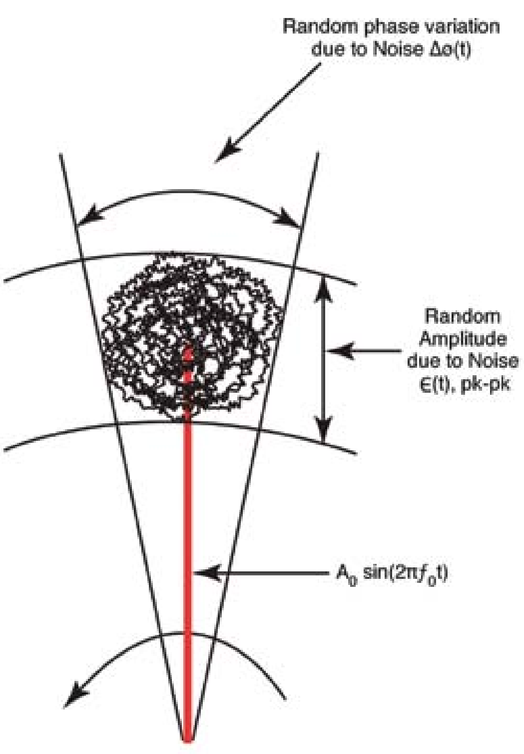

Fig. 3B Vector Representation of $V(t) = [A_0 + \epsilon(t)] \cdot \sin[2\pi f_0 t + \Delta\phi(t)]$ with Є(t) and Δø(t) random.

Figure 3B

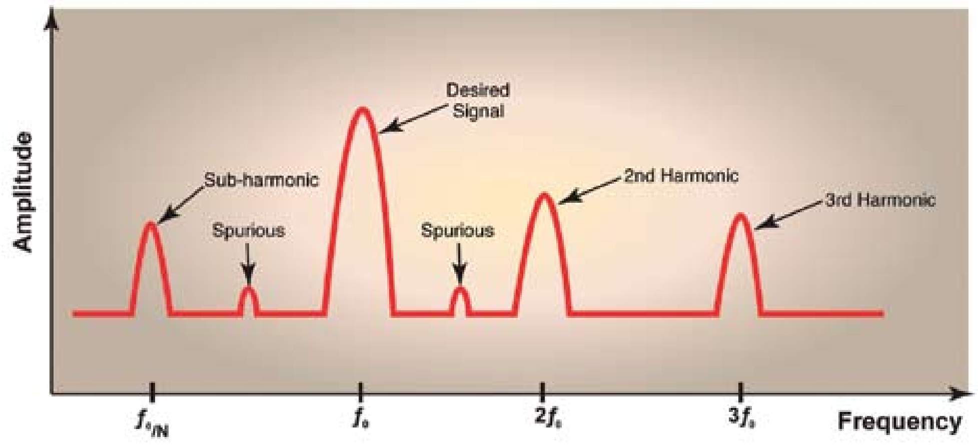

Figure 4: Typical output spectra of a real oscillator.

Conclusion

The output of an oscillator is not perfect. Due diligence must be conducted by the system engineer in specifying and validating performance of the oscillator correctly. Also, the system itself can easily corrupt an oscillator with either conducted or radiated signals. As experienced RF engineers know, it is best NOT to create/generate any un-wanted signals in the first place, rather than try to filter them somehow after they've been produced.

Glossary

Deterministic Jitter (DJ) - predictable jitter with a non-Gaussian probability density function (PDF). Deterministic jitter is always bounded in amplitude and with specific causes (e.g., intersymbol interference (ISI), duty cycle distortion (DCD), sub-harmonic(s) of the oscillator, etc.)

Phase Noise - the term most widely used to describe the characteristic randomness of frequency stability.

Spectral Purity - the ratio of signal-power-to-phase-noise sideband power.

Random Jitter (RJ) - Jitter that is not bounded and can be described by a Gaussian probability density function (PDF).

Thermal noise - noise generated by thermal agitation of electrons in a conductor.

Noise power is quantified as

$$P_n = kT\Delta f = kTB \text{ (watts)}$$

Where K is the Boltzmann's contant = 1.38×10^{-23}(J/K)

T is the absolute temperature in °K

And Δf= B is the *noise bandwidth* of the measurement system.